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Finite Time Observers for nonlinear system, Numerical example

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***Abstract*—In this paper, we consider the synthesis of Finite Time Observers for two class of nonlinear systems by solving some LMIs problem. First, we develop a method to calculate an affine gain which is used to determine an observer for a class of nonlinear systems. The method can also be applied to the case of another class of nonlinear systems with nonlinear output. A numerical example illustrates the proposed theory and point out the ameliorations comparing with asymptotic observers.**

***Index Terms*—Finite Time Observers, LMIs, nonlinear systems.**

I. INTRODUCTION

To master a process well, one should get a good knowledge about it. Generally, the variables that are directly measured all the susceptible sizes to describe this process and the states it can go though. In this way, we can deal with the problem of the reconstruction of the information which is not measured. Here, we are talking about the role of the observer or estimator of the state that consists on an auxiliary dynamic system where the inputs are the measure input/output of the process. This paper aims to present some indispensible and necessary notions about the construction of the observers for a non linear system. Moreover, it is going to deal with new approaches to construction of observers in terms of finite time for different classes of non linear systems. Finite Time Stability (FTS) is a concept which deals with systems which operation is limited to a fixed finite interval of time and for which, from practical considerations, the system's variables must lie within specific bounds. Different from another notion with the same name [3], [4] which deals with fast convergence, FTS is the only meaningful definition of stability for such systems [12]. Many fields in automatic control include this type of systems like robotic control [16], control of space vehicles and missiles, [10], chemical processes [10] and neural networks [5]. When dealing with observation of systems one is often concerned with asymptotic observers [1], [3], [4], [5], [12], [15] and [17]. But when it is required to guarantee FTS for a considered system, it is clear that we need an alternative notion of observers that we can call Finite Time Observers (FTO). The idea of an asymptotic observer is to design an observer such as the estimation error, which is the difference between real state and its estimate, is asymptotically stable, in other words it tends to zero when the time converges to infinity. Now, if it is important to guarantee that system variables dosen't exceed some bounds during a finite time interval it is necessary to propose an observer such as the estimation error evolve in a certain limits during the fixed interval of time. In [7], the authors propose a method to determine a finite time observer for linear systems. In this work, we are motivated by the fact that it may not exist any work in the literature in relation of the synthesis of observers for nonlinear systems.

The paper is organised as follows: section 2 introduces some preliminaries and states the problem we want to solve. Section 3 presents the first main result which allows the design of a finite time observer based on the calculus of an affine gain. An example illustrates the proposed approach and emphasized the contribution. Section 4 is devoted to another class of nonlinear systems with nonlinear output. In this case a finite time observer is proposed. A short conclusion ends the paper.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. Observer state for nonlinear systems

An observatory is an auxiliary dynamic system (O) where the inputs are the measured inputs/outputs system (S) where the output is partial piece of information about its states. The idea is presented in figure 1.

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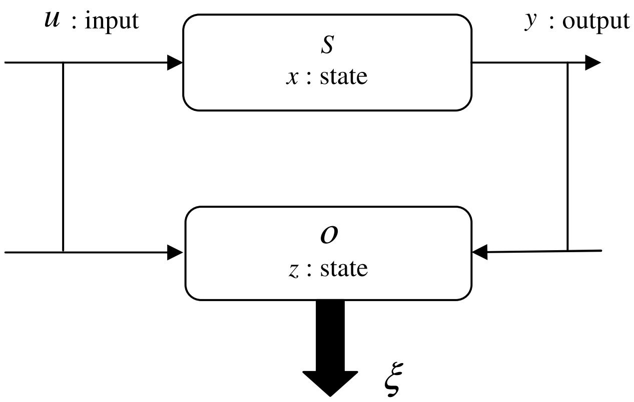

Fig. 1. *Principle of state estimation*

Definition 1 (S) is the dynamic system that is described as:

$$\dot{x} = f(x, u) \tag{1}$$

The system (o) is a local asymptotic observer for the system (S) if the following two conditions are checked:

1. $x(0) = \hat{x}(0) \Rightarrow x(t) = \hat{x}(t) \quad \forall\, t \geq 0;$

2. There is an open set neighbourhood $\Omega \subseteq \mathbb{R}^n$ of the origin as :

$x(0) - \hat{x}(0) \in \Omega \Rightarrow \|x(t) - \hat{x}(t)\| \to 0$ when $t \to +\infty$

If $\|x(t) - \hat{x}(t)\|$ tends exponentially toward zero, the system (o) is said exponential observer of (S)

When we get $\Omega = \mathbb{R}^n$, the system (o) is said global observer of (S).

♦

The second condition signifies that the estimation error must be asymptotically stable. Whereas, the first condition signifies that if both the observer (o) and the system (S) have the same initial state, the estimated state of (o) will be equal to the real state of the system (S) all the time.

There are different types of observers among which we can list:

1. Those which are based on the non linear transformation methods. This technique consists on transforming a non linear system into a linear one while the state will be estimated by a Luenberger.
2. Extended observer: in this case, the calculation of the observer's gain is made up starting from a linearised model around an operating point. We can mention the case of Kalman's extended filter or Luenberger's extendes observer.
3. High gain observer: this type of observers is generally used for the Lipchilizian systems. It is called so due to the fact that the selected observer's gain is large enough to compensate the non linearity of the system.
4. Generalized Luenberger Observers (GLO): this is a new type of observers that was recently proposed for the class of the monotone systems. This new design is to add to Luenberger observer a second gain inside the non linear part of the system.
5. Observers based on the contraction theory: this type of observers is based on the contraction theory which is used as a tool to analyze the convergence. This technique leads to new synthesis conditions that are different from those provided by the previous techniques.

B. Preliminaries

Let us consider the following system

$$\dot{x} = f(x), \quad x \in \mathbb{R}^n \tag{2}$$

By referring to [14] we can give the following definition:

Definition 2 System (2) is finite time stable with respect to (S_I, S_A, T) if and only if:

$$x(t_0) \in S_I \quad \Rightarrow \quad x(t) \in S_A, \quad \forall\, t \in [t_0, t_0 + T] \tag{3}$$

where S_I is the set of initial states and S_A is the set of admissible states.

♦

Definition 2 is stated in a relatively general way and it can be restricted to some interesting more tractable cases. For example, consider the following ellipsoidal compact sets.

$$S_I = \{x \in \mathbb{R}^n : x^T R\, x \leq c_1,\ c_1 > 0\} \tag{4}$$

$$S_A = \{x \in \mathbb{R}^n : x^T R\, x \leq c_2,\ c_1 < c_2\} \tag{5}$$

In such case the finite time stability (FTS) implies the satisfaction of relation (3) with constraints on initial state $x(t_0)$ and the actual state $x(t)$ defined by relations (4) and (5) respectively. In that case, we say that the system is finite time stable with respect to (c_1, c_2, R, T) and we will refer to the (c_1, c_2, R, T)-stability property.

The FTS means that the trajectory of the state emanating from an initial condition (taken in an ellipsoid defined by c_1 and R) remains in a given region (taken in an ellipsoid defined by c_2 and R) during a finite time interval (defined by T).

REMARK. 1— It is worth noting that the finite time stability notion introduced in this paper is different from that stated by some papers [6] and [7] and references therein, where it means fast convergence.

We have the following important result used in the next paragraphs and similar to the one presented in [10], [11].

Lemma 1 Let us define $V(x) = x^T R x$, and suppose that there exists a positive scalar β such that

$$\dot{V}(x) \le \beta V(x) \quad (6)$$

Then the system (2) is finite time stable with respect to (c_1, c_2, R, T) if the following inequality holds

$$\beta T < \ln\left(\frac{c_2}{c_1}\right) \quad (7)$$

We are now in position to define the problem we want to solve.

C. Problem formulation: Synthesis of the finite time observer

When there is a need to guarantee, for a considered system, the finite time stability, it is clear that we need an alternative notion of observers that we can call a finite time observers.

The asymptotic observer's idea is to design an observer so that the state error estimation is asymptotically stable. Now if we would like to guarantee that the system's state standard doesn't pass certain limits in a finite interval, it is necessary to offer an observer that the estimation error evolves in a certain limit during a giving time interval. Such observer is called finite time observer. In the majority of existing works dealing with nonlinear observers [1], [3], [4], [5], [12], [15] and [17], the observer is calculated such that the estimation error which is the difference between the real state and its estimate is asymptotically stable. In this context we can mention two drawbacks: In automatic control, the actions are achieved in a finite time and usually very short time. However, an asymptotically stabilizing control does not guarantee convergence performances. A second problem is related to the fact that estimation error can reach very important values enough to lead the estimate far away from the actual state which can be the origin of a violation of certain limits imposed to the state. Following these considerations and to go beyond the asymptotic context, we propose in this section, a finite time observer, allowing the stabilization in finite time of the estimation error. This design will force the tracking error to evolve within certain limits for a finite time interval corresponding to the time required to perform an operation. A new approach will be presented.

III. Approach based on affine gain

Consider the following control system

$$\dot{x}(t) = A x(t) + B f\left(x(t), y(t), u(t)\right) \quad (8)$$

$$y(t) = C x(t) \quad (9)$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control, $y \in \mathbb{R}^p$ is the output and A, B and C are constant matrices with appropriate dimensions. $f(x, y, u): \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^m \to \mathbb{R}^q$ is a differentiable function on x and satisfies the following hypothesis.

Hypothesis 1

$$a_{ij} \le \frac{\partial f_i}{\partial x_j}(x, y, u) \le b_{ij}, \quad \forall\, x, y, u \quad (10)$$

for all $i = 1,\dots,q$ and $j = 1,\dots,n$, with a_{ij} and b_{ij} are real constants.

Hypothesis 2

Let us suppose that there exists a set $S \subset \{1,\dots,q\} \times \{1,\dots,n\}$ such that

$$\frac{\partial f_i}{\partial x_j}(x, y, u) = g_{ij}(y, u) \ne 0, \quad \forall\,(i, j) \in S \quad (11)$$

REMARK. 2 — There exist many systems which satisfy condition (11) including the class of chaotic systems like Lorenz system [13] and Rössler system [22].

We consider a Luenberger observer in the form

$$\dot{\xi}(t) = A\xi(t) + B f\left(\xi(t), y(t), u(t)\right) + L\left(y(t), u(t)\right)\left(y(t) - C\xi(t)\right) \quad (12)$$

with

$$L\left(y(t), u(t)\right) = L_0 + \sum_{(i,j)\in S} g_{i,j}\left(y(t), u(t)\right) L_{i,j}.$$

We are interested by the design of matrices L_0 and $L_{i,j}$ which ensure that estimation error

$$\varepsilon(t) = x(t) - \xi(t) \quad (13)$$

is finite time stable with respect to given (c_1, c_2, R, T).

Definition 3 The observer (12) is said to be a finite time observer that is the estimation error (13) is Finite Time Stable with respect to (c_1, c_2, R, T) if and only if

$$\varepsilon_0^T R \varepsilon_0 \le c_1 \;\Rightarrow\; \varepsilon^T R \varepsilon \le c_2, \quad \forall\, t \in [0, T] \quad (14)$$

♦

The time derivative of error $\varepsilon(t)$ is given by

$$\dot{\varepsilon}(t) = \left(A - L\left(y(t), u(t)\right) C\right)\varepsilon(t) + B\,\delta f_t$$

or

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$$\dot{\varepsilon}(t) = \left(\tilde{A}(h(t)) - L(y(t),u(t))C\right)\varepsilon(t)$$

Where $\tilde{A}(h(t)) = A + B\sum_{i,j=1}^{q,n} h_{ij}(t) H_{i,j}^{q}$

with $h_{ij}(t) = g_{i,j}(y(t),u(t))$ for $i = 1,...,q$ and $j = 1,...,n$, $H_{i,j}^{q}$ and $h(t)$ are respectively given by

$$H_{i,j}^{q} = e_q(i)e_n^T(j) \quad \text{for } i = 1,...,q \text{ and } j = 1,...,n \quad (15)$$

$$h = (h_{11},...,h_{1n},...,h_{q1},...,h_{qn}) \quad (16)$$

The different parameters $h_{ij}(t)$ evolve in a bounded domain $H_{q,n}$. Let $\vartheta_{H_{q,n}}$ the set containing the $2^{q,n}$ vertices of $H_{q,n}$

$$\omega_{H_{q,n}} = \{\underline{\alpha}_{i,j} = \underline{g}_{i,j}, \overline{\alpha}_{i,j} = \overline{g}_{i,j} \text{ for } i = 1,...,q \text{ and } j = 1,...,n\}$$

where

$$\underline{g}_{i,j} = \min_t\left(g_{i,j}(y(t),u(t))\right)$$

$$\overline{g}_{i,j} = \max_t\left(g_{i,j}(y(t),u(t))\right)$$

Theorem 1
If there exist a symmetric positive definite matrix P, matrices R_0 and $R_{i,j}$ such that

$$\tilde{A}^T(\alpha)P - C^T\left(R_0 + \sum_{(i,j)\in S}\alpha R_{i,j}\right) + P\tilde{A} - \left(R_0 + \sum_{(i,j)\in S}\alpha R_{i,j}\right)C < \beta P, \quad \forall \alpha \in \omega_{H_{q,n}} \quad (17)$$

$$\beta T < \ln\left(\frac{c_2}{c_1}\right) \quad (18)$$

then the estimation error $\varepsilon(t)$ is finite time stable with respect to (c_1, c_2, P, T) and the matrices L_0 and $L_{i,j}$ are given by $L_0 = P^{-1}R_0{}^T$ and $L_{i,j} = P^{-1}R_{i,j}^T$.

Proof.
Let us consider the Lyapunov function

$$V(t) = \varepsilon^T P\,\varepsilon$$

we have

$$\dot{V}(t) = \varepsilon^T\,\psi(h(t))\,\varepsilon$$

where

$$\psi(h(t)) = \left(\tilde{A}(h(t)) - \left(L_0 + \sum_{(i,j)\in S} g_{ij}\,L_{ij}\right)C\right)^T P + P\left(\tilde{A}(h(t)) - \left(L_0{}^T + \sum_{(i,j)\in S} g_{ij}\,L_{ij}^T\right)C\right)$$

by lemma 1, the error is finite time stable with respect to (c_1, c_2, P, T) if the following condition is fulfilled $\dot{V}(t) \le \beta\,V(t)$, while satisfying the condition (18).

We know that

$$\psi(h(t)) \le \beta P \quad \text{for all } h_{ij} \in H_{q,n},\ i = 1,...,q,\ j = 1,...,n \ (18)$$

As ψ is affine in $h(t)$, (18) is satisfied if

$$\psi(\alpha) \le \beta P \quad \text{for all } \alpha \in \vartheta_{H_{q,n}}$$

What is true if (17) is satisfied and $R_0 = L_0^T P^T$ and $R_{ij} = L_{ij}^T P^T$.

Example 1 (Lorenz system [13])

Consider the system described by

$$\dot{x}(t) = A\,x(t) + B\,f(x) + Bg(x)\theta + B\,\eta(x,t)$$

with

$$A = \begin{pmatrix} -0.5 & -1 \\ 5 & -0.2 \end{pmatrix},\ B = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ C = \begin{pmatrix} 1 & 0 \end{pmatrix},$$

$$g(x) = \eta(x,t) = 0 \text{ and } f(x) = -5x_1 - 2x_2\sin(y).$$

we have

$$h_{i,j}(t) = \frac{\partial f_i}{\partial x_j}(z_i(t), y(t), u(t)),\ i = 1,\ j = 1,2$$

So $h_{11}(t) = -5$ and $h_{12}(t) = -2\sin(y(t))$. It can be shown easily that $\omega_{H_{q,n}} = \{-5, -2, 2\}$.

$$\sum_{i,j=1}^{1,2} h_{ij}(t)H_{i,j}^{1} = \begin{pmatrix} h_{11} & h_{12} \end{pmatrix}$$

$$\tilde{A}(h(t)) = \begin{pmatrix} h_{11}-1 & h_{12}+1 \\ 2 & -1.5 \end{pmatrix}$$

Given $c_1 = 0.35;\ c_2 = 3.5;\ T = 2.3s$. Our claim is to analyse FTS of the error estimation with respect to (c_1, c_2, P, T)

with

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$$P = \begin{bmatrix} 0,0578 & 0,0153 \\ 0,0153 & 0,8782 \end{bmatrix}.$$

Applying theorem 1, it is possible to synthesis an FTS observer with $\beta = 0.9$, $R_0 = [0.2880 \quad 9.2473]$, $R_{11} = [0 \quad -1.7897]$ and $R_{12} = [0 \quad 1.7897]$ implying $L_0 = [3.6881 \quad 4.8934]^T$, $L_{11} = [2.2524 \quad -0.9549]^T$ and $L_{12} = [-0.2524 \quad 0.9549]^T$.

Figure 2 shows that for 5 initial conditions such that $V(x) = \varepsilon_0^T P \varepsilon_0 < 0.35$, we have $V(x) = \varepsilon^T P \varepsilon < 3.5$ for $T < 2.3s$. Figure 3 and 4 show simultaneously real state and FTS estimate of the first and second component of the state.

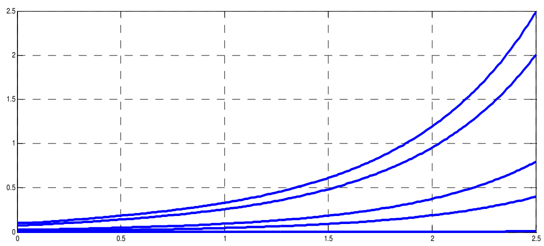

Fig. 2. *Evolution of V for 5 initial conditions such that* $V(x) = \varepsilon^T P \varepsilon < 0.35$

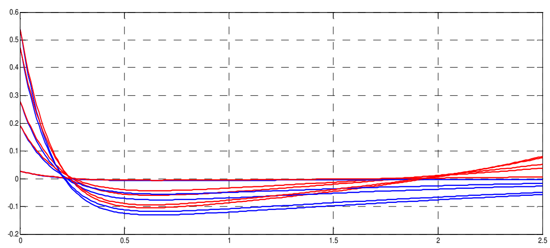

Fig. 3. *Evolution of* $x_1(t)$ *(red :estimate, blue :real)*

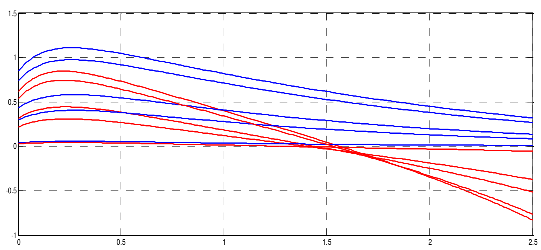

Fig.4. *Evolution of* $x_2(t)$ *(red :estimate, blue :real)*

IV. Approach based on nonlinear output

Let us consider the class of nonlinear systems with nonlinear output

$$\dot{x}(t) = A\,x(t) + B\,f\left(x(t), y(t), u(t)\right) \tag{19}$$

$$y(t) = g\left(x(t), u(t)\right) \tag{20}$$

where f satisfies hypothesis 1 and $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$ satisfies the following hypothesis

Hypothesis 3
Let us suppose that

$$\bar{a}_{ij} \le \frac{\partial g_i}{\partial x_j}(x,u) \le \bar{b}_{ij}, \quad \forall\, x, u$$

for all $i = 1,\dots,p$ and $j = 1,\dots,n$, with $\bar{a}_{ij}$ and $\bar{b}_{ij}$ are real constants.

We consider a Luenberger observer in the form

$$\begin{aligned}\dot{\xi}(t) = A\,\xi(t) + B\,f\left(\xi(t), y(t), u(t)\right) \\ + L\left(y(t) - g\left(\xi(t), u(t)\right)\right)\end{aligned} \tag{21}$$

We are interested by the design of a gain L which ensures that estimation error

$$\varepsilon(t) = x(t) - \xi(t)$$

is finite time stable.

The time derivative of ε is given by

$$\dot{\varepsilon}(t) = A\,\varepsilon(t) + B\,\delta f_t - L\,\delta g_t$$

where $\delta g_t = g\left(x(t), u(t)\right) - g\left(\xi(t), u(t)\right)$

Applying DMVT theorem [1] we conclude that there exist constants $v_i(t), i = 1,\dots,p$ such that :

$$\delta g_t = \left(\sum_{i,j=1}^{q,n} e_q(i) e_n^T(j) \frac{\partial g_i}{\partial x_j}\left(v_i(t), u(t)\right) \right) \varepsilon(t)$$

we denote by

$$\rho_{i,j}(t) = \frac{\partial g_i}{\partial x_j}\left(v_i(t), u(t)\right)$$

$$F_{i,j}^p = e_q(i) e_n^T(j) \quad \text{for} \quad i = 1,\dots,p \text{ and } j = 1,\dots,n \tag{22}$$

$$\rho = \left(\rho_{11},\dots,\rho_{1n},\dots,\rho_{p1},\dots,\rho_{pn}\right) \tag{23}$$

$$G\left(\rho(t)\right) = \sum_{i,j=1}^{p,n} \rho_{i,j}(t) F_{i,j}^p \tag{24}$$

we can write

$$\dot{\varepsilon}(t) = \left(\tilde{A}\left(h(t)\right) - L\,G\left(\rho(t)\right)\right)\varepsilon(t)$$

by hypothesis 3, the different parameters $\rho_{i,j}(t)$ evolve in a bounded domain $F_{p,n}$. Let $\vartheta_{F_{p,n}}$ the set containing the $2^{p,n}$ vertices of $F_{p,n}$

$$\vartheta_{F_{p,n}} = \left\{ \bar{\gamma}_{i,j} = \bar{a}_{i,j}, \underline{\gamma}_{i,j} = \bar{b}_{i,j} \quad \text{for } i = 1,\dots,p \text{ and } j = 1,\dots,n \right\}$$

Theorem 2

If there exist a symmetric positive definite matrix P , and a matrix R such that

$$\tilde{A}(\alpha)^T P - G(\gamma)^T R + P\,\tilde{A}(\alpha) - R^T G(\gamma) < \beta P, \quad \forall \alpha \in \vartheta_{H_{q,n}}, \forall \gamma \in \vartheta_{F_{p,n}} \tag{25}$$

$$\beta T < \ln\left(\frac{c_2}{c_1}\right) \tag{26}$$

Then the estimation error $\varepsilon(t)$ is finite time stable with respect to (c_1, c_2, P, T) and the estimation gain is given by $L = P^{-1}R^T$.

Proof

Let $P = P^T > 0$ and assume that there exists R which satisfies the condition (25).

Let the Lyapunov function

$$V(t) = \varepsilon^T P\, \varepsilon .$$

We have

$$\dot{V}(t) = \varepsilon^T\, \psi\big(h(t)\big)\, \varepsilon$$

where

$$\psi\big(h(t)\big) = \tilde{A}\big(h(t)\big)^T P - G\big(\rho(t)\big)^T L^T P + P\,\tilde{A}\big(h(t)\big) - P\,L\,G\big(\rho(t)\big)$$

Noting that $R = L^T P$, we obtain

$$\psi\big(h(t)\big) = \tilde{A}\big(h(t)\big)^T P - G\big(\rho(t)\big)^T R + P\,\tilde{A}\big(h(t)\big) - R^T\, G\big(\rho(t)\big)$$

By lemma 1, the error is finite time stable with respect to (c_1, c_2, P, T) if the following condition is fulfilled $\dot{V}(t) \le \beta\, V(t)$, while satisfying the condition (26).

We know that

$$\psi\big(h(t)\big) \le \beta\, P \quad \text{pour tout } h_{ij} \in H_{q,n}, \quad i = 1,\ldots,q,\; j = 1,\ldots,n \tag{27}$$

As ψ is affine in $h(t)$ while using the convexity principle [Weiss *et al.,* 1967], (27) is satisfied if

$$\psi(\alpha) \le \beta\, P \quad \text{for all } \alpha \in \vartheta_{H_{q,n}}$$

What is true if (25) is satisfied and $R = L^T P$.

V. Conclusion

For a general class of systems, we have seen that currently, there are no universal methods for the nonlinear observer synthesis. The approaches that have been developed to date represent either an extension of the algorithms that are used for linear systems (linearization around an operating point) or some specific algorithms for non linear systems or at least for certain classes. Thus, the design of finite time observers for a class of non linear systems has been studied. As they are based on solving some linear Matrix Inequalities, these techniques, for some classes of non linear systems that observation error remains within the limit for a given finite time interval. Furthermore, application to the manipulator arm has been used to highlight the potential of the proposed approaches.

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